Integration Types Checklist:

Hints for every type:

- First take any constants out of integral (must be a product to take out)
- Don't forget to write $+c$ after every indefinite integral (integral w/out limits)
- You can always use a substitution for every type instead (works for all types of $f(x)$ integrals). Let $u = f(x)$
- Differentiate answers to check if you've done the integration correctly

① Is it one of the standard forms?

yes

Powers:

1) $\int kx^n dx = \frac{kx^{n+1}}{n+1}, n \neq -1$

e.g. $\int 2x^5 dx$, $\int 2x(x+4)(x-2)dx$

2) $\int (ax+b)^n dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{(n+1)}$

e.g. $\int 6(2x-1)^5 dx$

3) $\int f'(x)(f(x))^n dx = \frac{(f(x))^{n+1}}{n+1}$

e.g. $\int 3x(x^2+2)^5$, $\int \cos x \sin^3 x\, dx$

Exponentials

1) $\int e^x dx = e^x$

2) $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b}$

e.g. $\int e^{4x} dx$

3) $\int f'(x) e^{f(x)} dx = e^{f(x)}$

e.g. $\int 4x\, e^{x^2+2} dx$

4) $\int a^x dx = \frac{1}{\ln a} a^x$

5) $\int a^{cx+b} dx = \frac{1}{c \ln a} a^{cx+b}$

e.g. $\int a^{3x-2} dx$

6) $\int f'(x) a^{f(x)} dx = \frac{1}{\ln a} a^{f(x)}$

e.g. $\int 3x\, a^{x^2+1} dx$

Natural log (ln)

1) $\int \frac{1}{x} dx = \ln|x|$

2) $\int \frac{1}{ax+b} dx = \frac{1}{a} ln|ax+b|$

e.g. $\int \frac{5}{2x-4} dx$

3) $\int \frac{f'(x)}{f(x)} dx = ln|f(x)|$

e.g. $\int \frac{3x}{x^2-4} dx$, $\int \frac{e^x}{e^x+2} dx$

Bring up and Power

1) $\int \frac{1}{x^n} dx = \int x^{-n} dx = \frac{x^{-n+1}}{-n+1}, n \neq 1$

e.g. $\int \frac{3}{x^5} dx$, $\int \left(\frac{2}{5x^2} + \frac{x}{2}\right) dx$

2) $\int \frac{f'(x)}{f(x)^n} dx \Rightarrow \int f'(x)(f(x))^{-n} dx = \frac{(f(x))^{-n+1}}{-n+1}$

e.g. $\int \frac{1}{(2x-4)^5} dx$, $\int \frac{3x^2}{\sqrt{6x^3+1}} dx$, $\int \frac{e^x}{\sqrt{e^x-1}} dx$

Basic Trigonometry

1) $\int \sin x\, dx = -\cos x$

2) $\int \sin(ax+b)\, dx = -\frac{1}{a}\cos(ax+b)$

e.g. $\int \sin 2x\, dx$ and $\int \sin(4x+1)\, dx$

3) $\int f'(x) \sin f(x)\, dx = -\cos f(x)$

e.g. $\int 5x \sin(3x^2) dx$ and $\int x^2 \sin(4x^3)\, dx$

4) $\int \cos x\, dx = \sin x$

5) $\int \cos(ax+b) dx = \frac{1}{a}\sin(ax+b)$

e.g. $\int \cos 3x\, dx$ and $\int \cos(2x+3)\, dx$

6) $\int f'(x) \cos f(x) dx = \sin f(x)$

e.g. $\int 3x \cos(2x^2)\, dx$ and $\int x^2 \cos(2x^3-2)\, dx$

7) $\int \sec^2 x\, dx = \tan x$.

8) $\int \sec^2(ax+b)\, dx = \frac{1}{a}\tan(ax+b)$

e.g. $\int \sec^2 2x\, dx$ and $\int \sec^2(4x+1)\, dx$

9) $\int f'(x)\sec^2 f(x)\, dx = \tan f(x)$

e.g. $\int x \sec^2(2x^2)\, dx$ and $\int x^2 \sec^2(4x^3+1)\, dx$

10) $\int \sec x \tan x\, dx = \sec x$

11) $\int \sec(ax+b)\tan(ax+b)\, dx = \frac{1}{a}\sec(ax+b)$

e.g. $\int \sec 2x \tan 2x\, dx$ and $\int \sec(2x-1)\tan(2x-1)\, dx$

12) $\int f'(x) \sec f(x) \tan f(x)\, dx = \sec f(x)$

e.g. $\int x \sec(2x^2)\tan(2x^2)\, dx$ and $\int x^2 \sec(x^3)\tan(x^3)\, dx$

13) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x$

14) $\int \operatorname{cosec}(ax+b)\cot(ax+b)\, dx = -\frac{1}{a}\operatorname{cosec}(ax+b)$

e.g. $\int \operatorname{cosec} 2x \cot 2x\, dx$ and $\int \operatorname{cosec}(3x+1)\cot(3x+1)dx$

15) $\int f'(x)\operatorname{cosec} f(x) \cot f(x)\, dx = -\operatorname{cosec} f(x)$

e.g. $\int 4x \operatorname{cosec} x^2 \cot x^2\, dx$ and $\int x^2 \operatorname{cosec} x^3 \cot x^3\, dx$

16) $\int \operatorname{cosec}^2 x\, dx = -\cot x$

17) $\int \operatorname{cosec}^2(ax+b)\, dx = -\frac{1}{a}\cot(ax+b)$

e.g. $\int \operatorname{cosec}^2 2x\, dx$ and $\int \operatorname{cosec}^2(4x-1)\, dx$

18) $\int f'(x)\operatorname{cosec}^2 f(x)\, dx = -\cot f(x)$

e.g. $\int x \operatorname{cosec}^2(4x^2)\, dx$ and $\int x^2 \operatorname{cosec}^2(7x^3)\, dx$

Extra common results:

$\int \tan x\, dx = \ln|\sec x|$

$\int \cot x\, dx = \ln|\sin x|$

$\int \sec x\, dx = \ln|\sec x + \tan x|$

$\int \operatorname{cosec} x\, dx = -\ln|\operatorname{cosec} x + \cot x|$

Inverse Trigonometry

1) $\int \frac{1}{\sqrt{a^2-(bx)^2}} dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)$

e.g. $\int \frac{1}{\sqrt{9-x^2}} dx$

2) $\int \frac{1}{\sqrt{a^2-(bx)^2}} dx = -\frac{1}{b}\cos^{-1}\left(\frac{bx}{a}\right)$

e.g. $\int \frac{-2}{\sqrt{9-16x^2}} dx$

3) $\int \frac{1}{a^2+(bx)^2} dx \Rightarrow \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$

e.g. $\int \frac{1}{4+25x^2} dx$

Hyperbolics (Further Maths only)

4) $\int \frac{1}{\sqrt{a^2+(bx)^2}} dx = \frac{1}{b}\sinh^{-1}\left(\frac{bx}{a}\right)$

e.g. $\int \frac{1}{\sqrt{9-x^2}} dx$

5) $\int \frac{1}{\sqrt{(bx)^2-a^2}} dx = \frac{1}{b}\cosh^{-1}\left(\frac{bx}{a}\right)$

e.g. $\int \frac{1}{\sqrt{9-x^2}} dx$

6) $\int \frac{1}{\sqrt{a^2-(bx)^2}} dx = \frac{1}{ab}\tanh^{-1}\left(\frac{bx}{a}\right)$

e.g. $\int \frac{1}{\sqrt{9-x^2}} dx$

Note: If none of the types above then either you must first do the following and then check back above

- Split fractions using rule $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

  $\int \frac{9x+6}{x} dx$, $\int \frac{(x+4)(x-1)}{3\sqrt{x}} dx$

- Factorise and cancel first

  $\int \frac{x+4}{2x^2+5x-12} dx$

- Divide first

  $\int \frac{x+2}{x-1} dx$

- Use partial fractions (may need to factorise the denominator or divide first)

  $\int \frac{5}{(x-1)(3x+2)} dx$, $\int \frac{5x^2-6x-3}{(x-1)(x^2-1)} dx$, $\int \frac{x^2-5x-6}{x^2-x-2} dx$

- Complete the square in the denominator

  $\int \frac{1}{x^2+6x+13} dx = \int \frac{1}{(x+3)^2+4} dx$

- Adapt the numerator of the fraction by splitting into 2 fractions

  $\int \frac{4x+19}{x^2+12x+41} dx = \int \left(\frac{4x+25}{x^2+12x+41} - \frac{5}{x^2+12x+41}\right) dx$

no

② Is it a fraction?

yes (see Natural log / Bring up and Power) — no

③ Is it a "harder trigonometry type"

yes

- An easy identity and then becomes standard form:

  $\int \cot^2 x dx = \int(\operatorname{cosec}^2 x - 1)dx$

  $\int \tan^2 x\, dx = \int(\sec^2 x - 1)dx$

  $\int \frac{1}{\sin^2 x} dx = \int \operatorname{cosec}^2 x\, dx$

  $\int \frac{1}{\cos^2 x} dx = \int \sec^2 x\, dx$

- A harder identity and then becomes standard form:

  $\int \sin^3 x dx = \int \sin x\,(1-\cos^2 x)\, dx$

  $= \int \sin x - \sin x \cos^2 x\, dx$

  $\int \sin^2 x\, dx = \int \frac{1-\cos 2x}{2} dx$

  $\int \cos^2 x\, dx = \int \frac{\cos 2x+1}{2} dx$

  $\int \sin^4 x\, dx = \int (\sin^2 x)^2 = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$

  $\int \cos^4 x = \int (\cos^2 x)^2 = \int \left(\frac{\cos 2x+1}{2}\right)^2 dx$

  $\int 15 \cos x \sin 2x\, dx = \frac{1}{2}\int(\cos 3x + \cos x)\, dx$

WATCH OUT for:

- Trig to a power (this is a standard form type mentioned already under in powers, but it is so often forgotten):

  $f'(x)(f(x))^n dx$ or $\int \frac{f(x)}{(f(x))^n} dx$

  $\int \sin x \cos^3 x\, dx$, $\int \frac{\sin x}{\sqrt[3]{\cos x}} dx$, $\frac{\sec^2 x}{\sqrt{\tan x}}$

- Fractions (this is a standard form type mentioned already under in fraction, natural ln, but it is so often forgotten):

  $\int \frac{f'(x)}{f(x)} dx$

  $\int \frac{\cos 2x}{\sin 2x + 4} dx$

  $\int \tan x dx = \int \frac{\sin x}{\cos x} dx$

  $\int \cot x dx = \int \frac{\cos x}{\sin x} dx$

- where you need to simplify a lot such as

  $\int \frac{\sin x}{\tan x} dx$, $\int \frac{\sqrt{1+\cos 2x}}{\sin x \sin 2x} dx$, $\int \frac{1-2\sin^2 x}{1+2\sin x \cos x} dx$

no

④ Is it a product of 2 unrelated distinct factors?

yes — no

⑤ By parts

$uv - \int v \frac{du}{dx} dx$

To know which u to choose: u is what comes first in LIATE

Ln

InvTrig

Algebra

Trig

Exp

e.g.

$\int xe^{4x} dx$

$\int x \sin 2x\, dx$

$\int x^2 \sin x\, dx$

$\int x^{\frac{1}{2}} \ln 2x dx$

$\int \sin 2x\, e^{4x}$

$\int \ln x\, dx$

$\int \arctan x\, dx$

Note: each time we use parts we "kill" a power of the algebra type. We sometimes have to do parts more than once

⑥ Substitution

(turns the integral a very simple standard form)

Substitutions to use:

Powers:

$\int \ldots (f(x))^n dx$

Let $u = f(x)$

Exponentials

$\int \ldots e^{f(x)} dx$

Let $u = f(x)$

Fractions

$\int \frac{\ldots}{(f(x))^1} dx$ or $\int \frac{\ldots}{(f(x))^n} dx$

Let $u = f(x)$

Trigonometry

$\int \ldots \sin(f(x))\, dx$

Let $u = f(x)$

Roots

$\int \frac{\ldots}{\sqrt{f(x)}} dx$ or $\int \ldots \sqrt{f(x)} dx$

Let $u = \cdots$

OR

$u^2 = f(x) \Leftrightarrow u = \sqrt{(f((x))}$

second sub is better

$\int \frac{\ldots}{a \pm \sqrt{f(x)}} dx$

Let $u = f(x)$

OR

$u = a \pm \sqrt{f(x)}$

second sub is better

Note: All Function $f(x)$ types on the left can use substitution instead: Let $u = f(x)$